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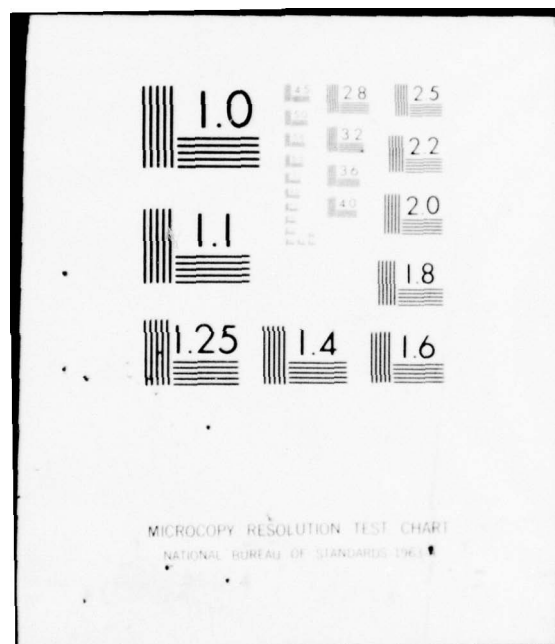
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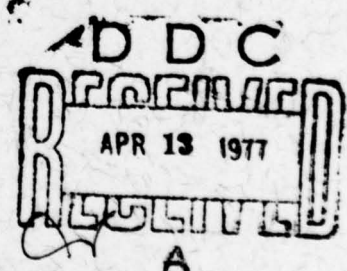
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Shock induced structural phase transformations, for two military motivated problems, are considered. Structural transformations in metallic alloys (e.g. TiNi) are examined from both macroscopic and microscopic (soft phonon mode) viewpoints. Both viewpoints are found to be consistent with the possibility of growth (avalanching) of the amplitude of a transformation inducing propagating shock wave. The relationship between such avalanching and the design considerations for an optically activated detonator is treated. (cont'd)		

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alpha to epsilon

The thermomechanics of the $\alpha \rightarrow \epsilon$ transition in iron (130 kilobars) is examined for the case where that transition is induced by a shock in the presence of an external magnetic field. Magnetostriction effects are included in the analysis. It is concluded that solid state physics relevant to the $\alpha \rightarrow \epsilon$ transition could be important to materials selection for armor.

alpha to epsilon

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I. INTRODUCTION

In this report we present some results of our investigations of structural phase transformations in media forced to experience time dependent density changes.

Our ultimate goals are in predicting strain and temperature amplitudes in army relevant materials which undergo structural phase transformations. While those ultimate goals have not yet been achieved, preliminary results give promise of some interesting physical interpretations, and novel military applications. We shall report on those preliminary results, and their possible applications, here.

Phase transformation physics has progressed from a mature and, to some, stale branch of thermodynamics, to the very lively study area of propagating correlated states^{1,2}. Those propagating correlated states can be described by constant shape solutions (called solitons³) of a nonlinear equation which models the interatomic potential and statistical mechanics of the transforming medium.

The approach taken here will be that of the middle ground. Rather than follow the soliton approach, which has not yet been applied to media of dynamically varying geometry (i.e. time dependent strain), we will proceed by incorporating some of the more modern aspects of phase change physics into a continuum mechanics framework.

Our interest in phase change phenomena stems from optical fuze train considerations. Picatinny Arsenal has an interest in developing an optically excited detonator containing only secondary explosives (for safety reasons). A typical design⁴ of an optically excited detonator is shown in Fig 1.

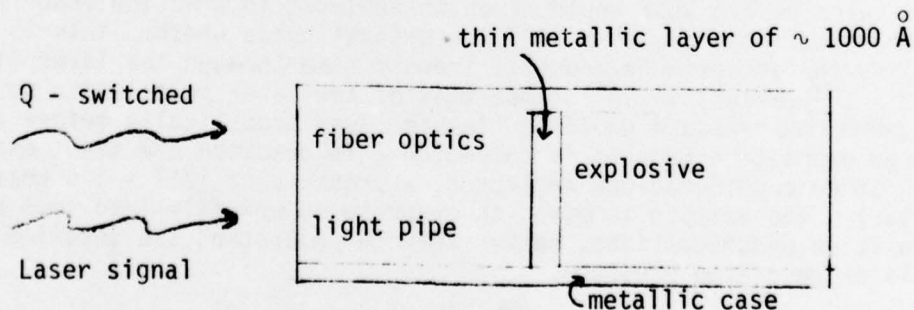


Figure 1. Typical design of an optically excited detonator.

In that design the thin metallic layer (e.g. aluminum) absorbs the burst (approximately 25 nanosecond pulse width) of optical energy, gets heated beyond its vaporization temperature, and expands. That expansion causes a compressive shock wave to enter the explosive resulting in initiation of detonation.

Unfortunately, the power requirements of the optical detonator system, illustrated in Fig. 1, for an explosive such as RDX, are approximately ^{4,5} 4J/cm² (25 nanosecond pulse width), making the design of a small (and thus small system mobile) laser initiation system difficult.

Consider the thin metallic layer shown in Fig. 1 for the case of optical energy deposition small compared to the metal's vaporization energy. In this case, the pressure induced into the explosive is proportional to the Gruneisen parameter, Γ , of the metal ($\Gamma \equiv \partial P / \partial \epsilon$ with ϵ the thermal energy density). Imagine the layer to be 10⁻⁴ cm thick, 1 cm in diameter, and to possess a typical Γ of 2. For an absorbed fluence of 10⁻² J/cm² (10⁵ ergs/cm²) the cgs pressure generated in the metal is given by*

$$P = \Gamma \epsilon = 2 \frac{10^5}{\frac{\pi}{4} (1)^2 10^{-4}} \text{ cgs} = 2.6 \times 10^9 \frac{\text{dynes}}{\text{cm}^2}, \quad (1)$$

or $P = 2.6 \text{ kbar}$ (1 bar $\equiv 10^6 \text{ dynes/cm}^2$). For a specific heat of 0.215 cal/gm^oK (0.90 J/gm^oK), corresponding to aluminum, the absorbed fluence would result in a temperature rise of

$$\Delta T = \frac{10^{-2}}{0.90} \left[2.7 \frac{\pi}{4} (1)^2 10^{-4} \right]^{-1} \text{ } ^\circ\text{K} = 52^\circ\text{K}, \quad (2)$$

*The value of 2.6 kbar could never be achieved in practice under the stated conditions and 25 x 10⁻⁹ sec optical pulse width. This is because the approximate acoustic transit time through the layer (10⁻⁴ cm / 5 x 10⁵ cm/sec) is 10⁻² times that of the laser pulse width so that the generated pressure would be "leaked" away acoustically before the energy deposition process is completed (in practice one might expect, with other considerations neglected, a pressure of 10⁻² x 2.6 kbar or 26 bar). The example is given in order to numerically lead into phase transition considerations, rather than to understand the detailed physics of laser generated pressure.

where we have used a density of 2.7 gm/cc (aluminum) in the calculation.

The 2.6 kbar pressure of Eq. (1), when associated with a pressure pulse of less than 10^{-6} sec duration, is incapable⁶ of initiating detonation in secondary explosives. The minimum pressure required is of the order of 20 kbar.

In Fig. 2 we show Grüneisen parameter for TiNi as a function of temperature. The curve was obtained⁷ by measuring the thermal expansion

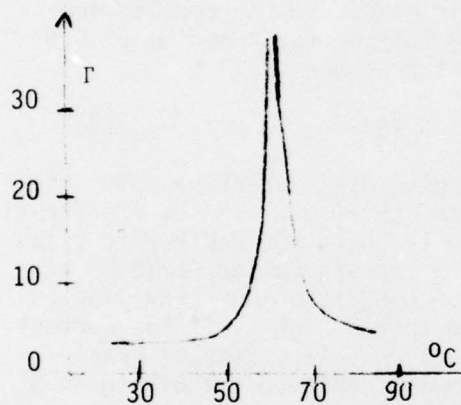


Figure 2. Grüneisen parameter vs temperature for TiNi. From Pace and Saunders⁷.

coefficient while the TiNi alloy was undergoing a structural transformation at approximately 60°C and atmospheric pressure (during a cooling cycle). Clearly a Grüneisen parameter value associated with the transition peak in Fig. 2, when substituted into Eq. (1), would meet our minimum pressure requirements for initiation of detonation in secondary explosives. The difficulty is that the large Γ values occur over a temperature range of approximately 5°C while, from Eq. (2) the energy deposition is expected to result in a temperature change* of approximately 50°K. The implication being that the large Γ values are only effective over a small fraction of the energy deposition process.

The discussion contained in the above paragraph illustrates the essentials of our first involvement with structural phase transformation physics for shock wave related fuze train⁹ applications. Clearly, while there is promise for an enhancement effect for small energy depositions (so that the resulting temperature change does not carry one beyond the transition region), the effects discussed so far do not offer any promise of say enhancing 2 kbar to 20 kbar.

There is another approach to be taken. That approach is, however, much more difficult, and requires a deep understanding of phase transformation physics. Materials such as TiNi, when they undergo a phase

* The specific heat also undergoes large variations in magnitude over a small temperature range in the region of the transition temperature⁸. Because those variations occur over so narrow a temperature range, the discussion based upon the 50°K result of Eq. (2) remains valid.

transformation either give up energy to, or absorb energy from the surroundings. That energy is known as the latent heat of transformation, and for TiNi (51 atomic percent Ni) the latent heat is 5.8 Cal/gm (exothermic on cooling)^{9,10}. Now the question becomes can we pump (i.e. force) the transformation with a shock wave in such a way that the liberated heat of transformation serves to reinforce the shock that served to trigger the transformation. A similar situation occurs with the detonation process in explosives - except in that case the integrated heat of transformation is of the order of 10^3 Cal/gm.

Figure 3 illustrates what we have in mind. Energy density and pressure have the same units. Thus 5.8 Cal/gm, for a medium of 6.5 gm/cc (NiTi), is equivalent to a thermal pressure of

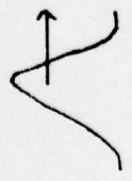
$$5.8 \frac{\text{Cal}}{\text{gm}} = 38 \frac{\text{Cal}}{\text{cm}^3} = 160 \frac{\text{J}}{\text{cm}^3} = 1.6 \text{ kbar.} \quad (3)$$

Consider a slab of NiTi alloy, and suppose that a pressure pulse of amplitude 0.5×1.6 kbar is traveling to the right. In the differential volume A assume that the pressure pulse triggers the exothermic transformation. The result of that triggered transformation could be a transformation induced pressure pulse of 0.5×1.6 kbar traveling to the left, and a similar pulse traveling to the right. If the correct conditions prevail the two right propagating pulses (the original pulse plus the transformation pulse) could reinforce, resulting in a net pressure pulse of 1.6 kbar. An adjacent differential volume could give similar reinforcement - the net result being a pressure pulse of avalanching amplitude.

There are a number of conditions which must be fulfilled in order for the avalanching process to proceed. While we will not now dwell on those conditions, let us simply mention that transformation induction time and the post transformation thermal equilibrium relaxation time are physical parameters which are important to the avalanching conditions.

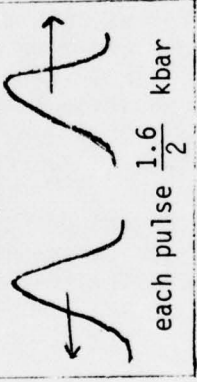
There are a number of other potential military applications of structural phase transformations. One such application involves the 130 kbar transition^{11,12} in iron alloys of low silicon and iron content. That transformation offers the possibility of tailoring armor (or impacting fragments) so as to control free surface spallation for offensive or defensive purposes; by controlling a transformation relaxation parameter one can apparently control¹¹ the sharpness of the unloading (rarefaction) shock and consequently, to some degree, control spall behavior. We shall comment further upon such applications, and the relevant transformation physics, within the main body of this report.

input pulse
of $\frac{1.6}{2}$ kbar



differential volume A

transformation
induced pulses



each pulse $\frac{1.6}{2}$ kbar

net pulse of 1.6 kbar
propagating to the
right.

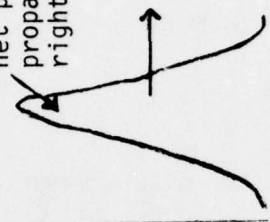


Figure 3. Schematic of the transformation induced avalanching process.

II. CONTINUUM CONSIDERATIONS

In this section we will consider some continuum aspects of static and dynamic phase transitions. We will begin by studying a relatively poor, but instructive, model for a transition forced by a propagating strain. Our main interest will be in further evaluation of the possibility of reinforcement avalanching as illustrated in Fig. 3.

Consider a one-dimensional problem characterized by

$$\rho_0 \frac{\partial^2 u}{\partial t^2} - c(x,t) \frac{\partial^2 u}{\partial x^2} = f(x,t), \quad (4)$$

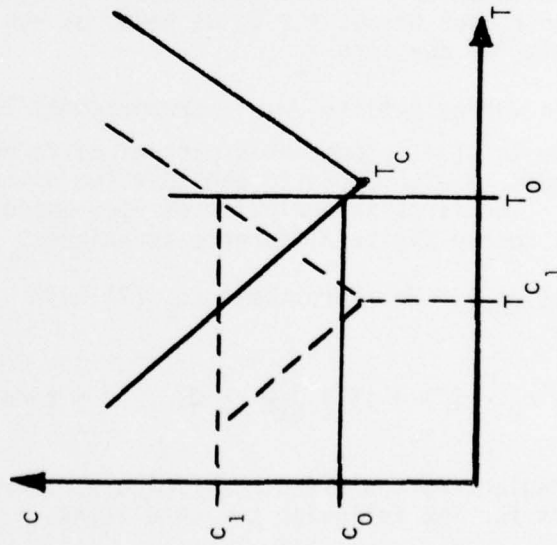
$$c(x,t) = c_0 + \alpha(x,t) [c_1 - c_0], \quad (5)$$

where ρ is mass density, u is particle displacement, c is an elastic constant, f denotes a viscosity functional, and the subscripts zero and one denote pre and post transformation states respectively. α can take on numerical values between one and zero and is meant to model the physics of the transformation. In writing Eq.(4) we have assumed an ideal* second order¹³ phase transition so that there is no volume discontinuity associated with the transition, and with the consequence that the mass density reference state is ρ_0 regardless of the phase.

Figure 4 gives an idea of the intent of Eqs. (4) and (5). It is known from experiment¹⁴ that tension raises the transition temperature (and consequently compression lowers the transition temperature) in TiNi.

*Ideal second order transformations are characterized by volume and entropy changing continuously through the transition. The result of that continuity is zero volume change and zero latent heat in going through the transition. Although we have seen that the NiTi transformation does possess a non-zero latent heat we will assume, for the purposes of modeling with Eq. (4), that the volume changes continuously.

Effect of compression. Initial temperature, T_0 , assumed less than T_c (at atmospheric pressure).



Effect of expansion. Initial temperature, T_0 , assumed greater than T_c (at atmospheric pressure).

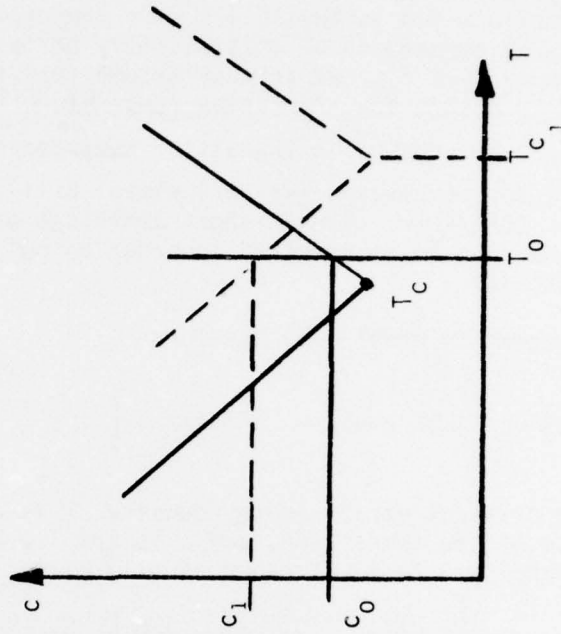


Figure 4. Change in elastic constants resulting from a stress induced transition (via a shift in transition temperature).

In our physics picture we allow, for example, a compression pulse to enter the solid which is initially at a temperature T_0 . The temperature T_0 is assumed to be only slightly below the zero stress transition temperature T_C . We further assume that the compression is strong enough to change the transition temperature to a value T_{C1} , where $T_{C1} < T_0$. The change in transition temperature with stress is very dramatic; for 55 percent (weight) nickel NiTi alloy, experiments¹⁴ show $\Delta T/\Delta P \sim 15^\circ\text{C/kbar}$. Thus a shock amplitude of only a few kilobars is all that should be required in order to pump a near room temperature transition.

We now proceed to model α in the form

$$\alpha(x,t) = \left[1 - \exp \left\{ - \frac{\epsilon(x,t)t}{\epsilon_C \tau} \right\} \right], \quad (6)$$

where ϵ_C is a critical strain energy density, τ is a relaxation time characteristic of the transition, and ϵ is the impressed strain energy density. Expanding α to first order in (ϵt) gives

$$\rho_0 \frac{\partial^2 u}{\partial t^2} - c_0 \left[1 + \left(\frac{c_1}{c_0} - 1 \right) \frac{\epsilon t}{\epsilon_C \tau} \right] \frac{\partial^2 u}{\partial x^2} = f(x,t). \quad (7)$$

Even though it is known^{7,8} that attenuation (viscosity) is strong in the transition region we set $f = 0$; we have not yet considered possible physical models for non zero f .

The strain energy density, ϵ , is proportional to $\left(\frac{\partial u}{\partial x} \right)^2$, and serves to make Eq. (7) a formidable partial differential equation. As a consequence we are forced to consider two different approaches to solutions. The first is analytic and very approximate, and the second is via coarse finite difference techniques.

A. Approximation I. We approximate Eq. (7) with

$$\rho_0 \frac{\partial^2 u}{\partial t^2} - c_0 (1 + \beta t) \frac{\partial^2 u}{\partial x^2} = 0, \quad \beta = \text{const.} \quad (8)$$

Eq. (8) is, neglecting the effect of finite f , a reasonable approximation to the physics for the following two conditions:

a. The frequency, ω , associated with the strain is so large that $\omega\tau \gg 1$ holds. Under this condition we can argue that the transforming medium sees an rms average strain rather than individual peaks and troughs.

b. $\xi > \xi_C$, so that the strain effect on the transition has reached saturation and is of little further consequence. This condition requires that $t < \tau$ in order that the α approximation used in Eq. (7) remains valid.

Neither of the above two conditions will be satisfied by the subsequent algebraic manipulations, but as the result offers a hint of relevant material behavior we shall proceed.

Substitute $u(x,t) = T(t) \chi(x)$ in Eq. (8) with the result

$$\frac{1}{(1+\beta t)T} \frac{d^2 T}{dt^2} = -m^2 \omega_o^2 = \frac{c_o}{\rho_o \chi} \frac{d^2 \chi}{dx^2}, \quad (9)$$

where $-m^2 \omega_o^2$ is the separation constant. m is a dimensionless number and ω_o is a constant with units of angular velocity. T is a dimensionless functional while χ has the dimensions of length. Thus

$$\chi_m(x) = \chi_m \exp \left[\pm i m \omega_o x \sqrt{\frac{\rho_o}{c_o}} \right], \quad (10)$$

and the T part of Eq. (9) can be rewritten as

$$\frac{d^2 T}{dy^2} + \frac{m^2 \omega_o^2}{\beta^2} y T = 0 \quad (11)$$

with $y = (1+\beta t)$.

Equation (11) is Airy's equation and its solutions can be written in the form¹⁵

$$T_m(t) = A_m U_1(Y,1) + B_m U_2(Y,2), \quad (12)$$

where

$$Y \equiv \left(\frac{m^2 \omega_o^2}{\beta^2} \right)^{1/3} (1+\beta t), \quad (13)$$

with U_1 and U_2 being linearly independent and tabulated¹⁵. The solution $u(x,t)$ is constructed by summing the product $T_m(t)\chi_m(x)$ over m .

Let us now evaluate β for the special exaggerated case of $c_1 = 2c_0$.
We also allow $\xi = \xi_c$.

$$\left\{ \begin{array}{l} c_1 = 2c_0 \\ \xi = \xi_c \end{array} \right\} \Rightarrow \beta = \frac{1}{\tau}, \quad (14)$$

$$\gamma = (m^2 \omega_0^2 \tau^2)^{1/3} (1 + t/\tau). \quad (15)$$

From the left hand side of Eq. (9) we see

$$\left. \frac{d^2 T_m}{dt^2} \right|_0 = -m^2 \omega_0^2, \quad (16)$$

so that if we identify ω_0 as the angular frequency of the applied disturbance (evaluated at $t=0$) there is justification* for considering only the $m=1$ contribution to $u(x,t)$. We thus proceed with the subscript m dropped.

* The amplitudes A_m and B_m are determined by the orthogonality of the set of functions of Eq. (10). Let

$$u(x,t) = \sum_n T_n(t) \chi_n(x).$$

Multiply both sides by $\exp \left[\pm i m \omega_0 x \sqrt{\frac{\rho_0}{c_0}} \right]$, and integrate

$$\text{over } x \quad \int_{-\infty}^{\infty} e^{\pm i m \omega_0 x \sqrt{\frac{\rho_0}{c_0}}} u(x,t) dx = 2\pi \sum_n T_n(t) \chi_m \delta(n-m).$$

Thus

$$T_m(0) \chi_m = \frac{1}{2\pi} \int_{-\infty}^{\infty} e^{\pm i m \omega_0 x \sqrt{\frac{\rho_0}{c_0}}} u(x,0) dx.$$

The above result implies that, for an ideal disturbance which at $t=0$ is a pure sinusoid (of angular velocity ω_0) filling all space, the solutions are characterized by A_1 and B_1 being the only amplitudes which do not vanish identically. While that ideal case can never be achieved in practice (in a typical experiment the $t=0$ disturbance resides only within the boundaries of a transducer), the ideal case serves as the foundation for concentrating our attention only on the $m=1$ component of the solution.

From Eqs. (10), (12), (13), and (15)

$$u(0,0) = AU_1 \left[(\omega_o \tau)^{2/3}, 1 \right] + BU_2 \left[(\omega_o \tau)^{2/3}, 1 \right], \quad (17)$$

$$\begin{aligned} \left. \frac{\partial u(0,t)}{\partial t} \right|_0 &= \frac{A}{\tau} (\omega_o \tau)^{2/3} U_1' \left[(\omega_o \tau)^{2/3}, 1 \right] + \\ &+ \frac{B}{\tau} (\omega_o \tau)^{2/3} U_2' \left[(\omega_o \tau)^{2/3}, 1 \right]. \end{aligned} \quad (18)$$

The U functions have the properties¹⁵

$$U_1(0,1) = 1, \quad U_2(0,1) = 0, \quad (19)$$

$$U_1'(0,1) = 0, \quad U_2'(0,1) = 1. \quad (20)$$

We now make the "hydrodynamic" approximation of $(\omega_o \tau) \ll 1$ (which is contrary to the condition \textcircled{a} which follows Eq. (3)) so that from Eqs. (17) and (19)

$$u(0,0) = A, \quad (21)$$

and from Eqs. (18) and (20)

$$\left. \frac{\partial u(0,t)}{\partial t} \right|_0 = \frac{B}{\tau} (\omega_o \tau)^{2/3}. \quad (22)$$

It has already been assumed, see Eq. (16), that the disturbance may be characterized at $(x,t) = (0,0)$ by

$$\left. \frac{\partial u(0,t)}{\partial t} \right|_0 \approx \omega_o u(0,0). \quad (23)$$

Eq. (23) is a rise time statement rather than a requirement that the applied disturbance be a plane wave sinusoid. Thus

$$u(0,t) = u(0,0) \left[U_1(\gamma,1) + (\omega_o \tau)^{1/3} U_2(\gamma,1) \right], \quad (24)$$

which predicts that in the hydrodynamic approximation $u(0,t)$ has the time dependence of $U_1(Y,1)$. Figure 5 displays that time behavior. From Eq. (15) we see that, for $t \geq 0$, our interest resides in $Y > 0$ for $c_1 = 2c_0$.

In order to judge the importance of the physics modeled by Eq. (8) we must compare displacements with and without the presence of the modeled phase transition. That comparison can be achieved by assuming

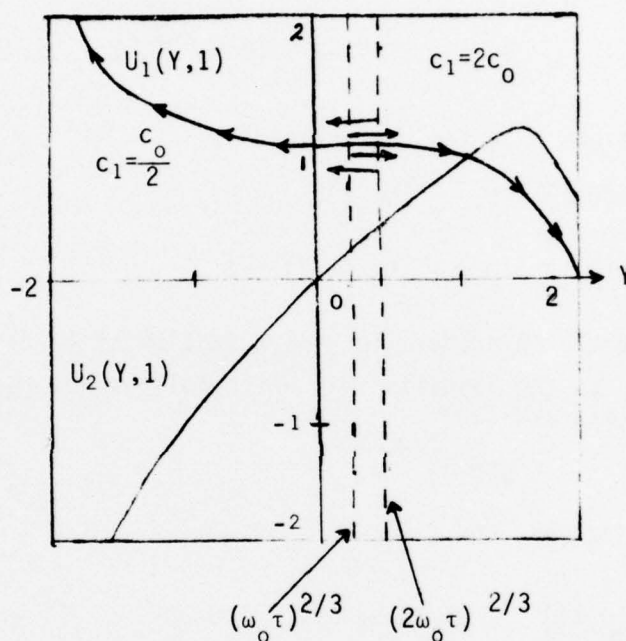


Figure 5. $U_1(Y,1)$ and $U_2(Y,1)$ vs Y .
 $(\omega_0 \tau)^{2/3}$ corresponds to $t=0$ for $c_1 = 2c_0$,
and $(2\omega_0 \tau)^{2/3}$ corresponds to $t=0$ for
 $c_0 = 2c_1$.

$$U(0,t) = u(0,0) \cos \omega_0 t \quad (25)$$

in the absence of a transition, and inquiring as to the value of t (which we call t_1) at which $U_1(Y,1)$ achieves its first node. From Eq. (15) with $m=1$, and Fig. 5,

$$(\omega_0 \tau)^{2/3} \left(1 + \frac{t_1}{\tau}\right) = 2 \quad (26)$$

For $(\omega_0 \tau) = 0.1$ we thus have

$$1 + 10 (\omega_0 t_1) = \frac{2}{(0.1)^{2/3}} \Rightarrow t_1 = \frac{0.83}{\omega_0} \quad (27)$$

t_1 compares to a time of $\pi/2\omega_0 = 1.57\omega_0$ for the node to be achieved in the absence of a transition. That difference in time increases as $(\omega_0 \tau)$ decreases.

The above analysis shows that the modeled transition serves to accelerate changes in the displacement $u(o,t)$. That acceleration may be viewed as a rudimentary form of avalanching. For the $c_1 = 2c_0$ case that avalanching is limited by the oscillatory behavior of $U_2(Y,1)$ for $Y > 0$. It should also be noted that the result of Eq. (27) violates the condition ⑤ (which follows Eq. (8)). That condition was important for interpreting the physics appropriate to the model of Eq. (8).

One can also consider the (exaggerated) elastic constant change of $c_1 = c_0/2$. Such a change can be obtained by requiring (see Fig. 4)

$$\left| \frac{T_0 - T_c}{T_0 - T_{c_1}} \right| > 1 \Rightarrow c_1 = \frac{c_0}{2} \quad (28)$$

rather than

$$\left| \frac{T_0 - T_c}{T_0 - T_{c_1}} \right| < 1 \Rightarrow c_1 = 2c_0 \quad (29)$$

as illustrated in the two examples of Fig. 4. For the change $c_1 = c_0/2$, Y and $u(o,t)$ become¹⁶, subject to the same assumptions as in the discussion of $c_1 = 2c_0$,

$$Y = (2\omega_0 \tau)^{2/3} \left(1 - \frac{t}{2\tau}\right), \quad (30)$$

$$u(o,t) = u(o,0) \left[U_1(Y,1) - (2\omega_0 \tau)^{1/3} U_2(Y,1) \right] \quad (31)$$

Equations (30) and (31) correspond to decreasing Y for increasing t . Here the avalanching effect is quite dramatic (see Fig. 5) in that $U_1(Y,1)$ is unbounded for $Y < 0$. As with Eq. (24), Eq. (31) suffers in that its use violates conditions ③ and ④ which follow Eq. (8).

However, regardless of the violation of those conditions, the above results hint at a type avalanching behavior consistent with Fig.(3).

B. Approximation II. We now will attempt to apply crude finite difference techniques to Eq. (7) with $f = 0$. Our objective being to gain physical insight beyond that capable with Eq. (8).

In Eq. (7) with $f = 0$ substitute $v_0^2 \equiv c_0/\rho_0$, $v_1^2 \equiv c_1/\rho_0$, $\xi \equiv M \left(\frac{\partial u}{\partial x} \right)^2$, and $\epsilon_c \equiv M\eta_c^2$. M is to be identified as an elastic constant, η_c a critical strain value, v_1 the longitudinal sound velocity in the transformed medium, and v_0 is to be identified as the pre-transformation longitudinal sound velocity. The result of the substitutions is

$$\frac{\partial^2 u}{\partial t^2} = v_0^2 \frac{\partial^2 u}{\partial x^2} + (v_1^2 - v_0^2) \left[\frac{t \left(\frac{\partial u}{\partial x} \right)^2}{\tau \eta_c^2} \right] \frac{\partial^2 u}{\partial x^2} \quad (32)$$

We now employ the so-called¹⁷ characteristic stretching transformation

$$\xi \equiv x - Vt, \quad \zeta \equiv \alpha Vt. \quad (33)$$

We further define

$$\psi(\xi, \zeta) = \frac{\partial u}{\partial \xi}. \quad (34)$$

The concept associated with Eqs. (33) is that of a disturbance characterized by an almost constant phase velocity V ; that is we seek solutions of Eq. (32) which have only weak ζ dependence (and conversely a strong ξ dependence). This is equivalent to assuming α small (and neglecting terms in α^2). If, for example, the non-linear term in Eq. (32) were zero, then a solution would be $\exp(ik\xi)$ with k constant and α identically zero. In other words we seek solutions which are only slightly removed from plane waves.

Substituting Eqs. (33) and (34) into Eq. (32), with α^2 terms neglected, gives

$$2\alpha V^2 \psi_\zeta + \left[(v_0^2 - V^2) + (v_1^2 - v_0^2) \left(\frac{\zeta \psi^2}{\zeta_a \eta_c^2} \right) \right] \psi_\xi = 0, \quad (35)$$

where

$$\zeta_a \equiv \alpha V \tau. \quad (36)$$

Straight forward finite differencing of Eq. (35) results in

$$\left[\frac{\Psi(\xi_n, \zeta_{m+1}) - \Psi(\xi_n, \zeta_m)}{\delta} \right] = \left[F_1 - F_2 \zeta_m \Psi^2(\xi_n, \zeta_m) \right] \left[\frac{\Psi(\xi_{n+1}, \zeta_m) - \Psi(\xi_n, \zeta_m)}{\Delta} \right], \quad (37)$$

where

$$F_1 \equiv \frac{V^2 - v_0^2}{2\alpha V^2}, \text{ and } F_2 \equiv \frac{v_1^2 - v_0^2}{2\alpha V^2 \zeta_a n_c^2} \quad (38)$$

δ and Δ are mesh dimensions and are defined by

$$\zeta_{m+1} = \zeta_m + \delta, \text{ and } \xi_{n+1} = \xi_n + \Delta. \quad (39)$$

Rewriting Eq. (37) with the time derivative single stepped backwards gives

$$\begin{aligned} \Psi(n, m) - \Psi(n, m-1) = & \left[G_1 - G_2 \zeta_m \Psi^2(n, m) \right] \Psi(n+1, m) + \\ & - \left[G_1 - G_2 \zeta_m \Psi^2(n, m) \right] \Psi(n, m), \end{aligned} \quad (40)$$

where

$$G_1 \equiv \frac{\delta}{\Delta} F_1 \quad \text{and} \quad G_2 \equiv \frac{\delta}{\Delta} F_2. \quad (41)$$

Thus, solving Eq. (40) for $\Psi(n+1, 1) - \Psi(n, 1)$

$$\Psi(n+1, 1) - \Psi(n, 1) = \frac{\Psi(n, 1) - \Psi(n, 0)}{G_1 - G_2 \zeta_1 \Psi^2(n, 1)}. \quad (42)$$

If the disturbance enters the transforming medium from the left at $(x, t) = (0, 0)$, then the $t=0$ strain, $\frac{\partial u}{\partial x}$, vanishes for $x > 0$. We thus set $\Psi(n, 0) = 0$ for $n > 0$. Furthermore, for $\tau \equiv m_a \delta$ and $\zeta_1 = \alpha V \delta$ Eq. (42) becomes for $n > 0$

$$\Psi(n+1,1) - \Psi(n,1) = \frac{\Psi(n,1)}{G_1} \left[1 - \frac{\Psi^2(n,1)}{m_a n_c^2} \frac{v_1^2 - v_o^2}{V^2 - v_o^2} \right]^{-1}. \quad (43)$$

For $\Psi^2 \lesssim n_c^2$ and $m_a > 1$, assuming $v_1^2 - v_o^2 \approx V^2 - v_o^2$, Eq. (43) can be approximated by

$$\Psi(n+1,1) - \Psi(n,1) \approx \frac{\Psi(n,1)}{G_1} \left[1 + \frac{\Psi^2(n,1)}{m_a n_c^2} \frac{v_1^2 - v_o^2}{V^2 - v_o^2} \right]. \quad (44)$$

Or

$$\begin{aligned} \Psi(n+1,1) \approx \Psi(n,1) + 2\alpha \left(\frac{\Delta}{\delta} \right) \left(\frac{V^2}{V^2 - v_o^2} \right) \Psi(n,1) + \\ + 2\alpha \left(\frac{\Delta}{\delta} \right) \frac{\Psi^3(n,1)}{m_a n_c^2} \left[\frac{V^2(v_1^2 - v_o^2)}{(V^2 - v_o^2)^2} \right] \end{aligned} \quad (45)$$

The second term on the right of Eq. (45) gives a purely coordinate system dependent linear strain contribution to the space dependent strain growth. The third term on the right contains the transformation physics (again coordinate system dependent because of the presence of α) contribution to the spatial strain growth. The third term is cubic in the strain, positive for $c_1 = 2c_o$, and negative for $c_1 = c_o/2$.

Equation (45) predicts, for $t < \tau$ (which allows the approximation of Eq. (44)), strain as a function of position for a medium undergoing a strain pumped transformation. It is consistent with spatial avalanching in that it contains a cubic strain contribution to the strain growth. An intelligent choice for V could be an experimentally observed¹⁸ velocity for propagation of a dynamic transformation.

The two continuum approximations considered in this section hint that shock amplification can occur in the presence of a strain pumped structural transformation.

III. LATTICE CONSIDERATIONS

Solid-solid phase transformations occur via two possible microscopic mechanisms. Broadly stated the mechanism of transformation related atomic motion may be classified as either diffusive or diffusionless.

Diffusive transformations, such as that associated with the formation of pearlite in low carbon content iron alloys, are slow processes¹⁹. Conversely, diffusionless transformations, such as that associated

with martensite formation in quenched steels¹⁹, can occur quite rapidly^{18,19}. The near room temperature transformation in almost stoichiometric NiTi is of the diffusionless variety^{10,8} and is often called a "martensitic" transition^{10,8}.

A rough idea of the maximum transition time for a transformation to be important to shock wave physics can be obtained by considering the planar impact between two half-centimeter discs of identical material as shown in Fig. 6. The impact initially produces a compressive shock of pressure amplitude $\rho_0 V U_S/2$ (see Fig. 6 for definition of symbols), and that pressure is reduced to zero when²⁰ the reflected shock, of pressure amplitude $-\rho_0 V U_S/2$, uncovers a previously compressed differential volume. A differential volume is thus at pressure for a time t_1 where $t_1 \leq \ell_1/U_S$. Thus, to the extent that shock pressure triggers the transition, the transition must occur in a time less than t_1 . For $\ell_1 = 0.5\text{cm}$ and $U_S = 5 \times 10^5 \text{ cm/sec}$, $t_1 \leq 2\mu\text{sec}$.

Another gross criterion for shock pumping a transition, with consequent reinforcing of the original shock, can be stated. If the transformation can propagate at a velocity approximately equal to or greater than the shock velocity, then shock pumping with reinforcement is a possibility. If the shock transformation propagates at a velocity considerably less than the shock velocity, the transition may be pumped, but reinforcement probably can not occur.

It is of interest to compare diffusive transformation numbers with the above velocity criterion. For a concentration n the diffusion coefficient D is defined by Fick's equation

$$\frac{\Delta n}{\Delta t} = D \frac{\Delta}{\Delta x} \left(\frac{\Delta n}{\Delta x} \right) . \quad (46)$$

For a diffusive displacement mean free path ℓ and corresponding mean free time τ , Eq. (46) can be approximated by

$$\frac{n}{\tau} = D \frac{n}{\ell^2} \quad (47)$$

Thus, for a diffusion velocity V_D defined by $V_D = \ell/\tau$, we have

$$V_D = D/\ell$$

For a solid or liquid the relevant mean free path is approximately a lattice parameter ($3 \times 10^{-8}\text{cm}$), while for a gas minimum ℓ is identified as the mean molecular separation. For an STP gas $\ell \sim 3.5 \times 10^{-7}\text{cm}$.

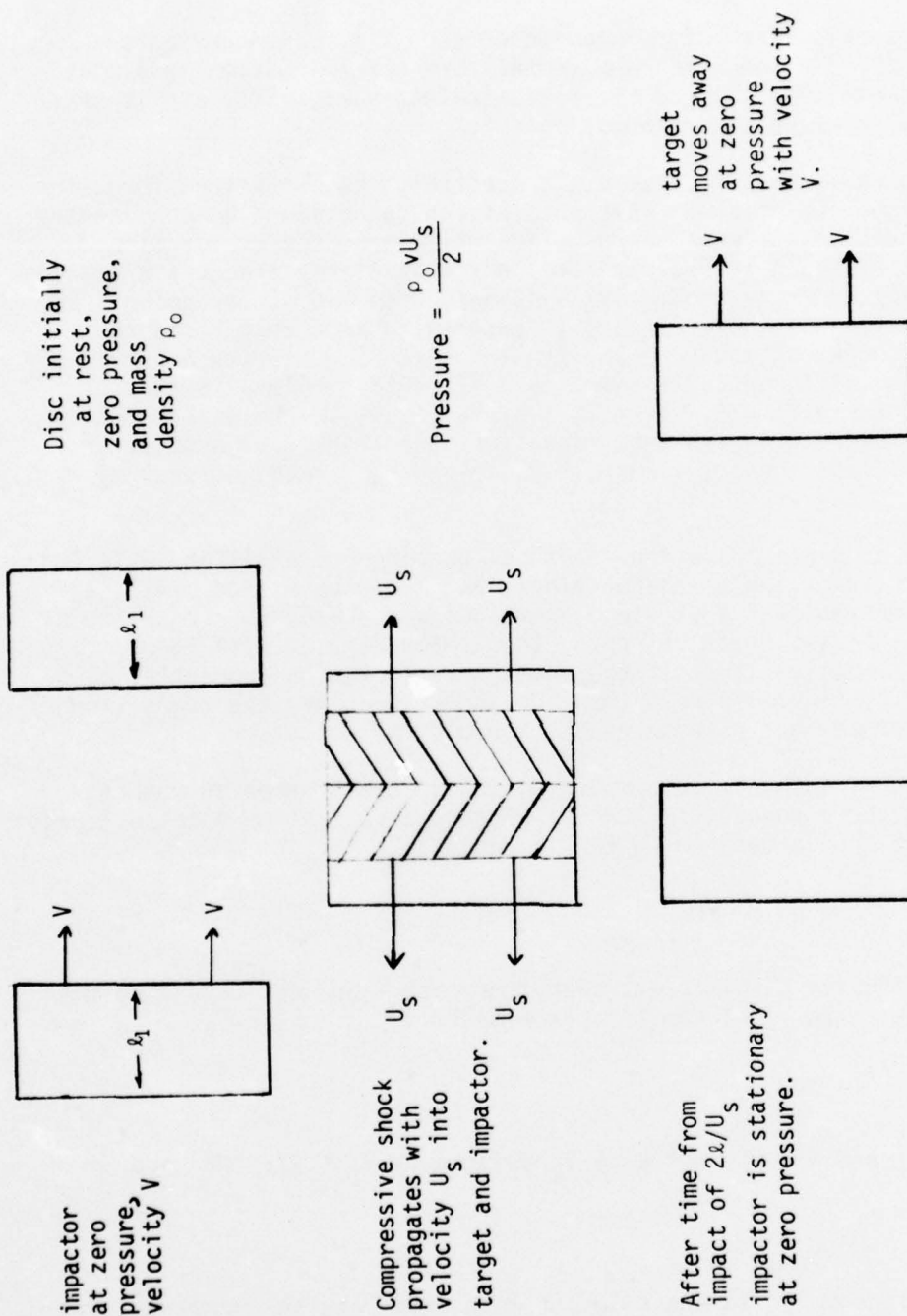


Figure 6. Typical shock wave generation by identical impact. (Compressive shock reflects from free surface as a rarefaction so that differential volume is pressurized for a time less than $2l_1/U_s$.)

Table 1 lists the three different ranges of the diffusion constants with the resulting diffusion velocities.

Table 1		
Diffusion Coefficient ^a and Velocity		
Substance	D	V _D
gas	0.2 cm ² /sec	5.7x10 ⁵ cm/sec
liquid	10 ⁻⁵	3x10 ²
solid	10 ⁻¹²	3x10 ⁻⁵

^a Diffusion coefficients obtained from American Inst. Phys. Handbook, D. E. Gray, editor (McGraw-Hill, New York, 1972). Third edition.

It is obvious, from Table 1, that shock pumped diffusive transformations in liquids and solids are not expected to give rise to reinforcement*. As our interest is in solids we will now consider some physics of diffusionless transformations.

Phonon Formulation. While diffusive processes are characterized by random atomic (or molecular) displacement, diffusionless processes are associated with coherent (the opposite of random) displacements. The displacements are coherent because there is a definite phase relationship between the displacive transitions of neighboring atoms.

The thermal vibrations of the atomic constituents of a crystalline (lattice) solid can be described^{21,22} by a set of independent harmonic oscillator states. The degree of occupancy (and the corresponding vibration amplitude) of each state is a function of the oscillator

* Gases, with STP acoustic velocities in the region of 3.3x10⁴cm/sec, would appear to be prime candidates for reinforcement. Perhaps that is the reason why explosive detonation in gaseous mixtures is so easy to achieve.

frequency and the temperature. The smallest* change in occupancy of a given state is called a phonon ^{21,22}. The elementary (i.e. differential) oscillations of each phonon in a given state are exactly in phase (i.e. coherent), and summing over the different states gives the net vibration of the crystal with respect to its center of mass.

The displacement χ of a simple harmonic oscillator of mass m obeys the differential equation

$$\frac{d^2\chi}{dt^2} + \omega^2\chi = 0, \quad \omega^2 = \frac{K}{m}, \quad (49)$$

where $\omega = 2\pi f$ with f being the oscillator frequency, and K is the "spring constant." The spring constant can be expressed in terms of elastic constants which are in turn a function of crystal structure ^{23,24} (or equivalently a function of the interatomic potential). ω is a function of the state variable, and for an ideal solid has a functional dependence as shown in Fig. 7.

* A change in occupancy, $(\Delta N)_{\vec{q}}$, causes a change in state energy via

$$(\Delta E)_{\vec{q}} = (\Delta N)_{\vec{q}} hf(\vec{q})$$

where h is Plank's constant, and $f(\vec{q})$ is the oscillator frequency for the vector state \vec{q} . $(\Delta N)_{\vec{q}} = \pm 1$ defines the addition or

subtraction of a single phonon with respect to the vector state of \vec{q}

In this section we will be interested in shock wave effects in those lattice solids which have the property that $\omega \rightarrow 0$ as $T \rightarrow T_c$ and $q \rightarrow q_c$ for some critical temperature T_c , and a critical state vector q_c , where $T_c > 0$.

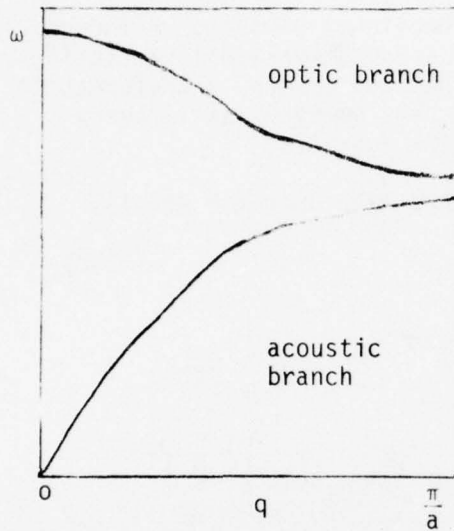


Figure 7. Frequency spectrum in $[100]$ direction for an ideal cubic lattice of unit cell dimension a .

Such behavior is related to a structural instability²⁵ (such as associated with a phase transition) and is not a property of an ideal lattice as illustrated in Fig. 7. A solid having the above property is said to possess a soft mode^{25,26}. There is a point of view²⁷ that all phase transformations, including the liquid \leftrightarrow gas transformation, can be described in terms of soft mode physics.

The "mode" $\omega(q)$ is said to be soft because, from Eq. (49), when $\omega \rightarrow 0$ the spring constant K also goes to zero (i.e. the spring becomes soft). As the vibrational restoring force is proportional to K , $K \rightarrow 0$ implies that the displacement does not return to zero. A displacement not returning to zero is conceptually equivalent to a structural transformation. For subsequent analysis we shall

employ the functional form²⁵

$$\omega^2(q) = A(T - T_c) + B(q - q_c)^2 \quad (50)$$

which exhibits the soft mode property.

We begin by considering a modified Fourier transform for the macroscopic displacement $u(x,t)$

$$u(x,t) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} G(q) N(q,x,t) u_q(t) e^{iqx} dq, \quad (51)$$

where $G(q)$ is a geometry dependent (density of states) factor, $N(q, x, t)$ is the previously mentioned occupancy of the state q , and $u_q(t)$ is the displacement amplitude. In principal $u_q(t)$ contains thermal vibrations (with a soft mode contribution), and also those displacements which constructively sum to give the center of mass motion necessary for successful continuum mechanics.

Equation (51) describes only one dimensional physics. Although structural transformations can occur on a one-dimensional lattice²⁸, Eq. (51) is limited in application in that the general transformation is explicitly three dimensional^{29*}. We can, however, still carry out some rather interesting and meaningful analysis.

Operate on $u(x, t)$ of Eq. (51) with the wave operator $W(x, t)$,

$$W(x, t) \equiv \frac{\partial^2}{\partial t^2} - \frac{c_0}{\rho_0} \frac{\partial^2}{\partial x^2}. \quad (52)$$

$$\begin{aligned} W(x, t)u(x, t) = & \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} G(q) \left[\left\{ \frac{\partial^2 N}{\partial t^2} - \frac{c_0}{\rho_0} \frac{\partial^2 N}{\partial x^2} \right\} u_q(t) + \right. \\ & + \left\{ 2 \frac{\partial N}{\partial t} \frac{du_q}{dt} - \frac{c_0}{\rho_0} (2iq) \frac{\partial N}{\partial x} u_q(t) \right\} + \\ & \left. + N(q, x, t) \left\{ \frac{d^2 u_q}{dt^2} + q^2 \frac{c_0}{\rho_0} u_q(t) \right\} \right] e^{iqx} dq. \quad (53) \end{aligned}$$

In the absence of interesting effects such as viscosity, non-linearity, or mode-softening, we would expect the right hand side to equal zero (corresponding to propagation of a simple disturbance with velocity $(c_0/\rho_0)^{1/2}$). Since, however, we are concerned with a particular phonon state q_c (the state with a softening spring constant) we can argue that to lowest order the physics of primary interest

* Structural transformations are usually discussed in terms of a discontinuity in the three dimensional symmetry transformation group (the set of transformations which leaves the structure, or its Hamiltonian equivalent unchanged).

resides only in the direction $\vec{q}_C/|\vec{q}_C|$. It is to be understood that Eq. (51), and the consequent analysis, has been written in that spirit.

The first curly bracket on the right in Eq. (53) represents second sound³⁰ (propagation of a disturbance in occupation number). Since pure second sound is not accompanied by a mass density disturbance³⁰, it will be naively assumed that the contents of the first curly bracket identically sum to zero. Any first curly bracket contribution to $Wu(x,t)$ awaits further work.

Now consider the $\partial N/\partial x$ contribution to the second curly bracket in Eq. (53).

$$\{Wu(x,t)\}_2 \equiv -i \left(\frac{2}{\pi}\right)^{1/2} \frac{c_0}{\rho_0} \int_{-\infty}^{\infty} q G(q) \frac{\partial N}{\partial x} u_q(t) e^{iqx} dq. \quad (54)$$

But in equilibrium

$$N(q,x,t) = \left[\exp\left(\frac{\hbar\omega}{kT}\right) - 1 \right]^{-1}, \quad (55)$$

$$\omega = \omega_0 + \Delta\omega, \quad T = T_0 + \Delta T, \quad (56)$$

where $\hbar \equiv h/2\pi$, k is Boltzmann's constant, and $\Delta\omega$ and ΔT are shock related changes in the mode angular frequency and local temperature. Using Eqs. (56) to expand Eq. (55) to first order in $\Delta\omega$ and ΔT gives

$$N \approx N_0 \left[1 + \frac{x_0 e^{x_0}}{e^{x_0} - 1} \left(\frac{\Delta T}{T_0} - \frac{\Delta\omega}{\omega_0} \right) \right], \quad (57)$$

where $x_0 \equiv \hbar\omega_0/kT_0$ and $N_0 = N_0(q)$ is the pre-shock occupation number.

Thus

$$\frac{\partial N}{\partial x} \approx x_0 e^{x_0} N_0^2 \left(\frac{1}{T_0} \frac{\partial T}{\partial x} - \frac{1}{\omega_0} \frac{\partial \omega}{\partial x} \right). \quad (58)$$

In phonon language the Grüneisen^{31,32} parameter $\Gamma(q)$ is defined by

$$(\Delta\omega(q)) \equiv -\Gamma(q)\omega_0(q)(\Delta\eta), \quad (59)$$

with η be the local strain ($\eta = \frac{\partial u}{\partial x}$). Substituting Eqs. (58) and (59) into Eq. (54) gives

$$\begin{aligned} \langle Wu(x,t) \rangle_2 = & -i \left(\frac{2}{\pi} \right)^{1/2} \frac{c_0}{\rho_0 T_0} \left(\frac{\partial T}{\partial x} \right) \int_{-\infty}^{\infty} q G(q) x_0 e^{x_0} N_0^2 u_q(t) e^{iqx} dq + \\ & -i \left(\frac{2}{\pi} \right)^{1/2} \frac{c_0}{\rho_0} \left(\frac{\partial \eta}{\partial x} \right) \int_{-\infty}^{\infty} q G(q) \Gamma(q) x_0 e^{x_0} N_0^2 u_q(t) e^{iqx} dq. \quad (60) \end{aligned}$$

For a mode independent Grüneisen parameter (i.e. $\Gamma(q) = \Gamma = \text{constant}$) one can show³³ that $(\Delta T)/T_0 = -\Gamma\eta$ with the result that $\langle Wu(x,t) \rangle_2 = 0$. For a transforming solid the assumption of constant Γ is particularly poor; in the introductory section (see Fig. 2) it was noted that Γ is a strong function of temperature in the vicinity of the transition temperature. Current thinking³⁴, at least for ferroelectrics, is that strong temperature dependence is related to the behavior of $\Gamma(q)$ near q_c . If we thus let

$$\Gamma(q) = \Gamma + (\Delta\Gamma)_q \quad (61)$$

in Eq. (60), and define

$$\langle (\Delta\Gamma)_q \rangle \equiv \int_{-\infty}^{\infty} q G(q) (\Delta\Gamma)_q x_0 e^{x_0} N_0^2 u_q(t) e^{iqx} dq, \quad (62)$$

Eq. (60) reduces to

$$\langle Wu(x,t) \rangle_2 = -\frac{g_2}{\rho_0} \frac{\partial T}{\partial x}, \quad (63)$$

where g_2 plays the role of a thermoelastic coupling coefficient.

$$g_2 = -i \left(\frac{2}{\pi} \right)^{1/2} c_0 \frac{\langle (\Delta\Gamma)_q \rangle}{\Gamma}. \quad (64)$$

The terms remaining to be considered in Eq. (53) can be grouped into what we shall call $\{Wu(x,t)\}_1$

$$\{Wu(x,t)\}_1 \equiv \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} G(q) N(q,x,t) e^{iqx} \otimes \left\{ \frac{d^2 u_q}{dt^2} + q^2 \frac{c_0}{\rho_0} u_q + \frac{2}{N} \frac{\partial N}{\partial t} \frac{du_q}{dt} \right\} dq. \quad (65)$$

As previously mentioned, each discrete q value defines an independent harmonic oscillator state

$$\frac{d^2 u_q}{dt^2} + \frac{1}{\tau_q} \frac{du_q}{dt} + \omega^2(q,T) u_q = 0, \quad (66)$$

where τ_q allows for the presence of damping²⁵. We have not explicitly included a strain dependent forcing term in Eq. (66). The idea is to implicitly include such an effect via the strain dependence of T and T_c through Eq. (50). Substituting Eq. (66) into Eq. (65) yields

$$\{Wu(x,t)\}_1 = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} G(q) N(q,x,t) e^{iqx} \otimes \left\{ \left(q^2 \frac{c_0}{\rho_0} - \omega^2 \right) u_q + \left(\frac{2}{N} \frac{\partial N}{\partial t} - \frac{1}{\tau_q} \right) \frac{du_q}{dt} \right\} dq. \quad (67)$$

The evaluation of Eq. (67) will begin with a rather drastic simplifying assumption. It will be assumed that Eq. (67) vanishes except near $q = q_c$, and that near q_c

$$\frac{1}{\tau_q} = \frac{2}{N} \frac{\partial N}{\partial t}. \quad (68)$$

Eq. (68) states that the phonon relaxation effects are just balanced by the dynamic strain induced phonon source term $\frac{\partial N}{\partial t}$. To claim other than Eq. (68) requires more detailed physics than is currently available to us. Thus, upon employing Eq. (50) at $q = q_c$, and Eq. (68), in Eq. (67) gives

$$\{Wu(x,t)\}_1 = \sqrt{\frac{(\Delta q)}{2\pi}} G(q_c) N(q_c) \left\{ q_c^2 \frac{c_0}{\rho_0} - A (T - T_c) \right\} u_{q_c} \otimes \otimes \left\{ e^{iq_c x} + e^{-iq_c x} \right\}, \quad (69)$$

where we have assumed* that G, N , and u_{q_c} are even function of q near $q = q_c$. (Δq) denotes a bandwidth centered around q_c .

The analysis which follows below utilizes $\frac{\partial}{\partial t} \{Wu(x,t)\}$.

If the further drastic assumption** $u_{q_c} = \text{const.} \equiv U_{q_c}$ is made, Eq. (69) reduces to

$$\frac{\partial}{\partial t} \{Wu(x,t)\}_1 = - F(q_c) \left\{ \frac{\partial T}{\partial t} - \frac{\partial T_c}{\partial t} \right\} \cos q_c x, \quad (70)$$

$$F(q_c) = (\Delta q) \left(\frac{2}{\pi} \right)^{1/2} G(q_c) N(q_c) A U_{q_c}. \quad (71)$$

* The assumption is trivial for G which happens to be a constant for one dimension geometry in q space. We have retained $G(q)$ in our calculations as a preliminary to eventual multi-dimensional calculations.

** This assumption means that u_{q_c} represents the correlated lattice displacement which results in a structural transformation.

A. Thermoelasticity with a Soft Mode. In a previous report³⁵ it was shown that thermoelasticity in the presence of thermal conductivity and longitudinal to transverse temperature (phonon occupation number) relaxation satisfies

$$\begin{aligned} (1 + \frac{\tau_1}{2} \frac{\partial}{\partial t}) (1 + \tau_1 \frac{\partial}{\partial t}) T - T + \frac{T_0 h_0 \tau_1}{2 g_0 \rho_0} (1 + \tau_1 \frac{\partial}{\partial t}) \frac{\partial v}{\partial x} = \\ = - \frac{v_0^2 \tau_1 \tau_2}{4} (1 + \tau_1 \frac{\partial}{\partial t}) \frac{\partial^2 t}{\partial x^2}, \end{aligned} \quad (72)$$

$$\text{where } v(x,t) \equiv \frac{\partial u(x,t)}{\partial t}, \quad n(x,t) \equiv - \frac{\partial u(x,t)}{\partial x}, \quad (73)$$

(notice the change in sign convention for n from that used earlier in this report).

$$\frac{\partial T_y}{\partial t} \equiv \frac{1}{\tau_1} (T_x - T_y), \quad T_y = T_z, \quad (74)$$

with T_y and T_z being the temperatures transverse to the x direction of strain propagation ($T_x \equiv T$), and

$$\frac{\partial T_x}{\partial t} \equiv \frac{2}{\tau_1} (T_y - T_x) + \frac{T_0 h_0}{\rho_0 g_0} \frac{\partial n}{\partial t} + \left(\frac{\partial T_x}{\partial t} \right)_{TC}, \quad (75)$$

which g_0 is given by $c_v/3$ (with c_v being the specific heat at constant volume) and $\left(\frac{\partial T_x}{\partial t} \right)_{TC}$ is the thermal conductivity contribution³⁵ to $\frac{\partial T_x}{\partial t}$.

$$\left(\frac{\partial T_x}{\partial t} \right)_{TC} \equiv - \frac{v_0^2 \tau_2}{2} \left(\frac{\partial^2 T_x}{\partial x^2} \right). \quad (76)$$

In Eqs. (72) and (76) $v_0 = (c_0/\rho_0)^{1/2}$.

Before proceeding with the analysis it remains to find the relationship between the thermoelastic coefficient h_0 (of Eqs. (72) and (75) and g_2 of Eq. (63). A linear thermoelastic medium has a coefficient h_0 defined by

$$\sigma = -c_0 \eta - h_0 T \quad (77)$$

where σ is stress. Thus the strain, η_{TE} , corresponding to stress free thermal expansion has the property

$$\frac{\partial \eta_{TE}}{\partial x} = - \frac{h_0}{c_0} \frac{\partial T}{\partial x} \quad (78)$$

But, from Eq. (63), considering time independent thermal expansion with $\eta = - \frac{\partial u}{\partial x}$

$$- \frac{c_0}{\rho_0} \frac{\partial^2 u}{\partial x^2} = \frac{c_0}{\rho_0} \frac{\partial \eta}{\partial x} = - \frac{g_2}{\rho_0} \frac{\partial T}{\partial x} \quad (79)$$

with the result that $h_0 = g_2$. Thus combining Eqs. (63) and (70) gives

$$(Wv) = - \frac{h_0}{\rho_0} \frac{\partial^2 T}{\partial t \partial x} - F(q_c) \left[\frac{\partial T}{\partial t} - \frac{\partial T_c}{\partial t} \right] \quad (80)$$

Proceeding now with the analysis by operating on Eq. (80) with $\frac{\partial}{\partial x}$.

$$\begin{aligned} W \frac{\partial v}{\partial x} = & - \frac{h_0}{\rho_0} \frac{\partial}{\partial t} \left(\frac{\partial^2 T}{\partial x^2} \right) - F(q_c) \left[\frac{\partial^2 T}{\partial x \partial t} - \frac{\partial^2 T_c}{\partial x \partial t} \right] \cos(q_c x) + \\ & + q_c F(q_c) \left[\frac{\partial T}{\partial t} - \frac{\partial T_c}{\partial t} \right] \sin(q_c x). \end{aligned} \quad (81)$$

Now operating with W of Eq. (52) on Eq. (72), and substituting from Eq. (81), gives

$$\begin{aligned}
 & W \left(1 + \frac{\tau_1}{2} \frac{\partial}{\partial t} \right) \left(1 + \tau_1 \frac{\partial}{\partial t} \right) - WT - \frac{T_0 h_0^2 \tau_1}{2 g_0 \rho_0} \left(1 + \tau_1 \frac{\partial}{\partial t} \right) \frac{\partial}{\partial t} \left(\frac{\partial^2 T}{\partial x^2} \right) + \\
 & - F(q_c) \frac{T_0 h_0 \tau_1}{2 \rho_0 g_0} \cos(q_c x) \left(1 + \tau_1 \frac{\partial}{\partial t} \right) \left[\frac{\partial^2 T}{\partial t \partial x} - \frac{\partial^2 T_c}{\partial t \partial x} \right] + \\
 & + q_c F(q_c) \frac{T_0 h_0 \tau_1}{2 \rho_0 g_0} \sin(q_c x) \left(1 + \tau_1 \frac{\partial}{\partial t} \right) \left[\frac{\partial T}{\partial t} - \frac{\partial T_c}{\partial t} \right] = \\
 & = - \frac{v_0^2 \tau_1 \tau_2}{4} W \left(1 + \tau_1 \frac{\partial}{\partial t} \right) \frac{\partial^2 T}{\partial t^2}. \quad (82)
 \end{aligned}$$

Expanding the $(1 + \tau_1 \frac{\partial}{\partial t})$ terms in Eq. (82) and collecting terms allows Eq. (82) to be rewritten as

$$\begin{aligned}
 & \frac{3}{2} \tau_1 W_4 \frac{\partial T}{\partial t} + \frac{\tau_1^2}{2} W_3 \frac{\partial^2 T}{\partial t^2} = - \frac{v_0^2 \tau_1 \tau_2}{4} W \frac{\partial^2 T}{\partial x^2} + \\
 & - \frac{v_0^2 \tau_1^2 \tau_2}{4} W \frac{\partial^2}{\partial x^2} \left(\frac{\partial T}{\partial t} \right) + F \frac{T_0 h_0 \tau_1}{2 g_0 \rho_0} \cos(q_c x) \left[\frac{\partial T}{\partial x \partial t} - \frac{\partial^2 T_c}{\partial x \partial t} \right] + \\
 & + F \frac{T_0 h_0 \tau_1^2}{2 g_0 \rho_0} \cos(q_c x) \left[\frac{\partial}{\partial x} \left(\frac{\partial^2 T}{\partial t^2} - \frac{\partial^2 T_c}{\partial t^2} \right) \right] + \\
 & - q_c F \frac{T_0 h_0 \tau_1}{2 g_0 \rho_0} \sin(q_c x) \left[\frac{\partial T}{\partial t} - \frac{\partial T_c}{\partial t} \right] \\
 & - q_c F \frac{T_0 h_0 \tau_1^2}{2 g_0 \rho_0} \sin(q_c x) \left[\frac{\partial^2 T}{\partial t^2} - \frac{\partial^2 T_c}{\partial t^2} \right], \quad (83)
 \end{aligned}$$

where

$$W_3 = \frac{\partial^2}{\partial t^2} - \left(\frac{c_0}{\rho_0} + \frac{T_0 h_0^2}{g_0 \rho_0} \right) \frac{\partial^2}{\partial x^2}, \quad (84)$$

$$W_4 = \frac{\partial^2}{\partial t^2} - \left(\frac{c_0}{\rho_0} + \frac{T_0 h_0^2}{3g_0 \rho_0} \right) \frac{\partial^2}{\partial x^2}. \quad (85)$$

If $(\omega_0/2\pi)$ is the Fourier frequency giving the dominant contribution to T , and if the limit $\omega_0 \tau \rightarrow 0$ (with $\tau = \tau_1$ or τ_2) is taken, then Eq. (83) may be approximated with

$$\frac{3}{2} \tau_1 W_4 \frac{\partial T}{\partial t} = F \frac{T_0 h_0 \tau_1}{2g_0 \rho_0} \left[\cos(q_c x) \left\{ \frac{\partial^2 T}{\partial x \partial t} - \frac{\partial^2 T_c}{\partial x \partial t} \right\} + \right. \\ \left. - q_c \sin(q_c x) \left\{ \frac{\partial T}{\partial t} - \frac{\partial T_c}{\partial t} \right\} \right]. \quad (86)$$

In evaluating the contributions from the first and second terms on the right of Eq. (83) we have employed the approximation $v_0^2 \frac{\partial^2}{\partial x^2} \sim \frac{\partial^2}{\partial t^2}$.

This last approximation assumes that the temperature profile is of the form

$$T(x,t) = T_1 \{ q(x - v_0 t) \} f(x,t) \quad (87)$$

where $\omega_0 \sim qv_0$ and f is a dimensionless function which varies slowly compared to T_1 . Eq. (87) is dominant Fourier term analysis in disguise, and is conceptually justified by picturing the temperature profile as approximately in step with the strain profile which in turn is propagating with a velocity nearly v_0 .

Employing Eq. (87) shows that the ratio of the first to second term on the right of Eq. (86) is given by (q/q_c) when the signs and cosines are taken as unity. Thus in the long wavelength limit (i.e. $q = 2\pi/\lambda$ goes to zero) we are left with

$$W_4 \frac{\partial T}{\partial t} = -q_c \frac{FT_0 h_0}{3g_0 \rho_0} \sin(q_c x) \left(\frac{\partial T}{\partial t} - \frac{\partial T_c}{\partial t} \right). \quad (88)$$

It is possible to show that, in the absence of a phase transformation (i.e. with $F=0$), the strain satisfies an equation which is one-to-one with the $F=0$ version of Eq. (83):

$$\begin{aligned} \frac{3}{2} \tau_1 W_4 \frac{\partial \eta}{\partial t} + \frac{\tau_1^2}{2} W_3 \frac{\partial^2 \eta}{\partial t^2} = \frac{v_0^2 \tau_1 \tau_2}{4} W \frac{\partial^2 \eta}{\partial x^2} + \\ - \frac{v_0^2 \tau_1 \tau_2}{4} W \frac{\partial^2}{\partial x^2} \left(\frac{\partial \eta}{\partial t} \right). \end{aligned} \quad (89)$$

Thus in the same $(\omega \tau) \rightarrow 0$ limit as used in arriving at Eq. (88) we have

$$W_4 \frac{\partial \eta}{\partial t} = 0 \quad (90)$$

in the absence of transformation physics. Comparing Eq. (88) and Eq. (90) suggests* that in the presence of transformation effects**

$$W_4 \frac{\partial \eta}{\partial t} = 0 + (\text{second order terms}). \quad (91)$$

Our transformation model, as illustrated in Fig. 4, has the transition temperature as a function only of the strain, $T_c = T_c(\eta)$.

Thus, from Eq. (91),

$$W_4 \frac{\partial T_c}{\partial t} = \frac{\partial \eta}{\partial t} W_4 \frac{\partial T_c}{\partial \eta} + \frac{\partial T_c}{\partial \eta} (\text{second order terms}). \quad (92)$$

* We have not yet been successful in deriving an explicit version of Eq. (91) with transformation effects included.

** If the left hand side of Eq. (88) is considered to be first order (because of the implicit presence of ΔT in $\frac{\partial T}{\partial t}$), then the right hand side is second order because of the product $F \frac{\partial T}{\partial t}$ (the function F contains the modal displacement u_q).

If the constitutive law $T_C = T_C(\eta)$ is now chosen of the form

$$\frac{\Delta T_C}{T_C^{(0)}} = \gamma \eta, \quad \gamma = \text{constant}, \quad (93)$$

in keeping with experimental observation¹⁴ for the room temperature transition in TiNi, then Eq. (92) becomes

$$W_4 \frac{\partial T_C}{\partial t} = 0 + \gamma T_C^{(0)} \text{ (second order terms)}, \quad (94)$$

with $T_C^{(0)}$ being the $\eta = 0$ value of T_C .

Experimentally¹⁴ $\gamma \sim 10$ for the TiNi transition[†] so that one can not, as a general statement, set the right hand side of Eq. (94) to zero compared to the right hand side of Eq. (88) unless $T_C^{(0)}$ itself is very small. We shall now make that $T_C^{(0)}$ small assumption in order to facilitate further analysis. This restriction to low temperature transformations represents a severe limitation on the theory presented in this report. Hopefully the results will at least hint at the behavior of room temperature transitions.

The assumption of Eq. (68) is of a different class than that of small $T_C^{(0)}$ in that it is possible that Eq. (68) holds for some room temperature transitions.

[†] γ , see the discussion associated with Eq. (61), is a Grüneisen-like parameter. That its magnitude is order ten, rather than of order unity as for non-transforming media³⁶, is probably very significant; that same magnitude of approximately ten characterizes the Grüneisen parameter in the region of a phase transition (see Fig. 2). The implication is that γ is determined by the physics of the mode q_C .

Letting $T_c^{(0)}$ be small in Eq. (94) gives

$$W_4 \frac{\partial T_c}{\partial t} = 0 \quad (95)$$

so that Eq. (88) can be written

$$W_4 \frac{\partial T_d}{\partial t} = - q_c \frac{FT_0 h_0}{3g_0 \rho_0} \sin(q_c x) \frac{\partial T_d}{\partial t}, \quad (96)$$

$$T_d \equiv T - T_c.$$

Now set

$$\tau_F^2 = \frac{3g_0 \rho_0}{q_c FT_0 h_0}, \text{ and } c_4^2 = \frac{c_0}{\rho_0} + \frac{T_0 h_0^2}{3g_0 \rho_0^2}, \quad (97)$$

so that Eq. (96) becomes

$$\frac{1}{c_4^2} \frac{\partial^2}{\partial t^2} \left(\frac{\partial T_d}{\partial t} \right) = \left[\frac{\partial^2}{\partial x^2} - \frac{1}{c_4^2 \tau_F^2} \sin(q_c x) \right] \frac{\partial T_d}{\partial t}. \quad (98)$$

Equation (98) is separable. Let $\frac{\partial T_d}{\partial t} \equiv T_1(t) \chi_1(x)$

with T_1 having the dimensions of $\frac{\partial T}{\partial t}$, and χ_1 dimensionless.

Thus

$$\frac{1}{c_4^2 T_1} \frac{\partial^2 T_1}{\partial t^2} = - \frac{m^2 \Omega^2}{c_4^2} = \frac{1}{\chi_1} \left[\frac{\partial^2}{\partial x^2} - \frac{1}{c_4^2 \tau_F^2} \sin(q_c x) \right] \chi_1, \quad (99)$$

where $m^2 \Omega^2 / c_4^2$ is the separation constant (Ω is a constant similar to ω_0).

$$\therefore \frac{d^2 x}{dx^2} + \left[\frac{m^2 \Omega^2}{c_4^2} - \frac{1}{c_4^2 \tau_F^2} \sin(q_c x) \right] x_1 = 0. \quad (100)$$

The Laplace transform, $x_L(s)$, of the function $x(x)$ is defined by

$$x_L(s) \equiv \int_0^\infty x(x) e^{-sx} dx. \quad (101)$$

Thus multiplying both sides of Eq. (101) by e^{-sx} and integrating over x gives*

$$\begin{aligned} \left(s^2 + \frac{m^2 \Omega^2}{c_4^2} \right) x_L(s) + \frac{i}{2c_4^2 \tau_F^2} \left[x_L(s - iq_c) - x_L(s + iq_c) \right] = \\ = \left. \frac{dx}{dx} \right|_0 + s x(0) \equiv BC, \end{aligned} \quad (102)$$

where the vertical slash with subscript zero indicates that the functional to the left is to be evaluated at $x=0$, and BC simply denotes the boundary condition functional.

If in Eq. (102) q_c is set equal to zero (i.e. $q_c=0$) the result is

$$\left(s^2 + \frac{m^2 \Omega^2}{c_4^2} \right) x_L(s) = BC \quad (103)$$

which, upon taking the inverse transform³⁷ of $x_L(s)$, is easily seen to result in propagation of $\frac{\partial T_d}{\partial t}$ amplitudes at constant phase velocity (without growth). That same result can be obtained directly from Eqs. (88) and (95).

* The second bracketed term on the left in Eq. (102) is obtained by employing $\sin(q_c x) = \frac{1}{2i} \{ \exp(iq_c x) - \exp(-iq_c x) \}$ prior to integration.

Now consider zone edge q_c (i.e. $q_c = \pi/a$ in Fig. 7). The inverse transform is given by³⁷

$$x_f(x) = \frac{1}{2\pi i} \int_{\gamma_c - i\infty}^{\gamma_c + i\infty} x_L(s) e^{sx} ds, \quad (104)$$

so that if $s \equiv ir$ we can limit the range of integration to $|r| \leq q_c$ (from Eq. (104) it is seen that r plays the role of q since r is conjugate to x , and $s \equiv ir$ transforms Eq. (104) into a Fourier transform if γ_c is chosen to equal zero). Now let $y = iq_c$.

$$\therefore x_L(s+iq_c) = x_L(y+s) = \sum_{n=0}^{\infty} \frac{s^n}{n!} \frac{d^n x_L(y)}{dy^n}, \quad (105)$$

$$x_L(s-iq_c) = x_L(-y+s) = \sum_{n=0}^{\infty} \frac{s^n}{n!} \frac{d^n x_L(-y)}{dy^n}. \quad (106)$$

Substituting Eqs. (105) and (106) into Eq. (102) gives

$$\begin{aligned} \left(s^2 + \frac{m^2 \Omega^2}{c_4^2}\right) x_L(s) + \frac{i}{2c_4^2 \tau_F^2} \sum_{n=0}^{\infty} \frac{s^n}{n!} \frac{d^n}{dy^n} \left[x_L(y) - x_L(-y) \right] = \\ = \left. \frac{dx_L}{dx} \right|_0 + s x_f(0) \equiv \text{BC}. \end{aligned} \quad (107)$$

If one attempts to take the inverse transform of Eq. (107) on a term by term basis, one sees³⁷ that the contributions from $n \geq 2$ in the summation are undefined. Thus, unless $x_L(y)$ is an even function y , Eq. (107) is consistent with the possibility of avalanching of the spatial part of $\frac{\partial T_d}{\partial t}$.

That the $q_c=0$ solution does not predict avalanching is not surprising. $q_c=0$ implies that the transformation is associated with

an infinite wavelength, and consequently that the transformation occurs simultaneously in the entire solid sample. Since the physics being discussed involves a strain disturbance propagating WITHIN a transformable medium, there is nothing present in the model being used which is capable of exciting the $q_c=0$ mode.

The above physics represents our start on the interplay between the microscopic and the macroscopic for strain pumped structural transformations. While the work is very incomplete it is supportive of the possibility of avalanching of the macroscopic parameter (e.g. strain and temperature).

Future work in this area will probably concentrate on deriving functionally explicit versions of Eqs. (91) and (94), and on solving equations similar to Eq. (102).

IV. THE $\alpha \rightarrow \epsilon$ TRANSITION IN IRON

Drummond³⁸, in 1957, predicted the possibility of a rarefaction (i.e. expansion) shock being produced by the unloading process in a medium which has been forced through a dynamic (compressive shock induced) phase transition. In the absence of such a phase transition (or other peculiar phenomena^{39,40}) the unloading process would produce a rarefaction fan*,^{41,42} rather than a shock.

The intersection of two rarefaction waves results in tension⁴³, and an adequate stress-time history^{44,45} in the tension regime can cause spallation (planar fracture). It has been shown by Duvall et al¹¹ that the relaxation time with which the phase change induced mass fraction approaches an equilibrium value has an important role in determining the rise time (stress-time history) of the rarefaction shock. Thus, since stress-time history is important to spallation, it is to be expected that control of phase change parameters will lead to some control over the satisfying of spallation criteria. Such control could be important to the degree of vulnerability of military armor.

* By fan we simply mean the stronger the unloading amplitude, the slower the propagation velocity associated with that amplitude.

Among the material parameters which are important to the Duvall et al relaxation time, percentage alloy (e.g. x in $\text{Ti}_x\text{Ni}_{1-x}$) and impurity (and defect) concentration are probably the most significant.

External (or applied) parameters probably also play an important role in the phase change spallation problem. The α -phase of iron is ferromagnetic, and one would thus expect the presence of an externally applied magnetic field to influence the transition. The analysis of the previous two sections involved the dominant Fourier amplitude of the applied strain disturbance. The relative strength of the strain induced Fourier amplitudes is a function of the rise time of the applied deformation, and that rise time can be influenced by impactor structure. A long rise time can, for example, be achieved employing an impactor of graded mass density (the low mass density region impacting first).

At this moment the details of the above spall control concepts are rather nebulous. There has, however, been some discussion in the literature concerning the effect of an applied magnetic field on the $\alpha \rightarrow \epsilon$ transition, and we will now concentrate on the physics relevant to that effect.

Curran and Hornmann^{46,47} have performed shock experiments which indicate that a magnetic field can reduce the dynamic transition pressure by approximately 10 kbar. Barker and Hollenbach subsequently performed a similar experiment⁴⁸, and found the transition pressure unchanged to within approximately 1 kbar. To make the situation even more intriguing, Curran and Hornemann^{46,47} have presented a derivation which predicts an increasing transition pressure with increasing magnetic field intensity.

We will now present a derivation which is identical in its essentials to that of Curran and Hornemann except that it includes a magnetoelastic⁴⁹ contribution.

Consider the Gibb's free energy⁵⁰, G , for a system composed initially (i.e. at atmospheric pressure) of α iron and in which the α and ϵ phases can exist in equilibrium at a pressure $P_c(H)$ where c denotes critical and H is the magnetic field intensity.

$$G = E - TS + PV \quad (108)$$

where E is the total internal energy of the system, and S the entropy.

$$\therefore dG = dE - TdS - SdT + PdV + VdP, \quad (109)$$

but, assuming a reversible process, from the first law of thermodynamics

$$TdS = dE + PdV \quad (110)$$

so that Eq. (109) becomes*

$$dG = - SdT + VdP. \quad (111)$$

If it is now assumed that the temperature changes are insignificant as compared to the pressure changes, then

$$G(P_C(H), T) - G(0, T) = \int_0^{P_C(H)} V(P, T, H) dP, \quad (112)$$

where we include a V dependence on H in order to allow for magnetoelasticity. The magnetoelasticity of interest here arises due to the interaction of H with the ferromagnetic structure of the α phase. The dependence of P_C on H only mirrors the fact that magnetostriction makes a contribution to the effective elastic constants. Thus the pressure necessary to achieve a given critical volume is magnetic field dependent.

* In writing Eqs. (108) through (111) we have implicitly included magnetic energy, proportional to H, in E. We have also assumed that the magnetic field frequencies of interest are so small that they do not make a separate pressure contribution and influence V through the electrical conductivity related penetration depth.

Now write Eq. (112) separately for each phase.

$$G_{\alpha}(P_C(H), T) - G_{\alpha}^{(H)}(0, T) = \int_0^{P_C(H)} V_{\alpha}(P, T, H) dP, \quad (113)$$

$$G_{\epsilon}(P_C(H), T) - G_{\epsilon}^{(H)}(0, T) = \int_0^{P_C(H)} V_{\epsilon}(P, T, H) dP, \quad (114)$$

where Eq. (114) represents an extension of the ϵ phase equation of state down to $P=0$. But a system of two phases (i.e. α and ϵ) which are in equilibrium at constant temperature and pressure (i.e. at constant $P_C(H)$) has the property⁵⁰

$$G_{\alpha}(P_C(H), T) = G_{\epsilon}(P_C(H), T). \quad (115)$$

Thus upon subtracting Eq. (114) from Eq. (113) while using Eq. (115) gives

$$- \left[G_{\alpha}^{(H)}(0, T) - G_{\epsilon}^{(H)}(0, T) \right] = \int_0^{P_C(H)} [V_{\alpha} - V_{\epsilon}] dP. \quad (116)$$

If the magnetic field contribution to $G^H(0, T)$ is treated as a perturbation, then

$$G^{(H)}(0, T) = G^{(0)}(0, T) + g(H), \quad (117)$$

where⁵¹

$$g(H) = - \frac{1}{2} HMV \quad (118)$$

In Eq. (118) M denotes the magnetic moment per unit volume, and the factor of $\frac{1}{2}$ arises from assuming that M is linear in H . Experimentally⁵² it is known that the magnetic moment of the ϵ phase is zero (i.e. the ϵ phase is not ferromagnetic) so that $g_{\epsilon} = 0$. Furthermore, for $H = 0$,

$$-\left[G_{\alpha}^{(0)}(0,T) - G_{\epsilon}^{(0)}(0,T) \right] = \int_0^{P_C^{(0)}} \left[V_{\alpha} - V_{\epsilon} \right]_{H=0} dP. \quad (119)$$

Thus, from Eqs. (116), (118), and (119)

$$\frac{1}{2} HMV = \int_0^{P_C^{(H)}} (\Delta V)_H dp - \int_0^{P_C^{(0)}} (\Delta V)_0 dP. \quad (120)$$

Equation (120) allows a solution for $\delta P \equiv P_C^{(H)} - P_C^{(0)}$. First $(\Delta V)_H$ is to be expanded to first order in H .

$$(\Delta V)_H = (\Delta V)_0 + \left. \frac{\partial (\Delta V)_H}{\partial H} \right|_0 H \quad (121)$$

with the result

$$\frac{1}{2} HMV \approx (\Delta V)_0 (\delta P) + H \int_0^{P_C^{(H)}} \left. \frac{\partial (\Delta V)_H}{\partial H} \right|_0 dP. \quad (122)$$

If, in Eq. (122), the partial derivative with respect to H is set equal to zero, one would essentially have the theoretical result of Curran and Hornemann^{46,47} with the consequence that δP would be predicted to be positive. Let δP_1 denote that value of δP .

$$\delta P_1 \left\{ \frac{(\Delta V)_0}{V_0} \right\} \equiv \frac{1}{2} HM. \quad (123)$$

δP_1 will now be evaluated using the following values for the relevant parameters in Eq. (123).

Volume terms⁵³: $v_{\alpha}(130 \text{ kb}, H=0) = 6.62 \frac{\text{cm}^3}{\text{mole}}$

$v_{\epsilon}(130 \text{ kb}, H=0) = 6.29 \frac{\text{cm}^3}{\text{mole}}$

$$v_0 \equiv \frac{1}{2} (v_{\alpha} + v_{\epsilon}), \quad (\Delta v)_0 \equiv (v_{\alpha} - v_{\epsilon}), \quad (124)$$

$$\therefore \frac{(\Delta V)_0}{V_0} \approx 5.11 \times 10^{-2}$$

Field Intensity⁴⁶: $H = 2000 \text{ Ampere-turns/meter}$

Magnetization (M): $M = K\mu_0 H$. (125)

In Mks units $\mu_0 = 4\pi \times 10^{-7} \text{ Henrys/meter}$, and K is the dimensionless susceptibility⁵⁴. Substituting the above in Eq. (123) gives

$$\delta P_1 = (4.9 \times 10) K \frac{\text{Newtons}}{\text{m}^2} = (4.9 \times 10^{-7}) K \text{ kbar}. \quad (126)$$

For α iron K has an approximate range of between 120 and 5×10^3 (depending upon previous magnetic history and H). A magnetic field intensity of 2000 Ampere-turns/m (equivalent to 25 Gauss) is small compared to the saturation magnetic field for iron of between 2×10^3 Gauss and 4×10^3 Gauss. Thus, while understanding that previous magnetic history is most important for small values of H , it seems most reasonable to use $K = 2 \times 10^3$ in Eq. (126), with the result

$$\delta P_1 \sim 10^{-3} \text{ kbar}. \quad (127)$$

The ϵ phase is not ferromagnetic, and its magnetostriction can thus be neglected. Additionally we will assume that the α phase magnetostriction is independent of stress, with the result that Eq. (122) can be approximated by

$$\frac{1}{2} HM \approx \frac{(\Delta V)_0}{V_0} (\delta P) + \frac{HP_C}{V_0} \left. \frac{\partial V_{\alpha}}{\partial H} \right|_0, \quad (128)$$

where for approximation purposes we have replaced $P_c(H)$ by $P_c = 130$ kbar. The problem now reduces to evaluating $\left. \frac{\partial V_\alpha}{\partial H} \right|_0$ from magnetostriction data.

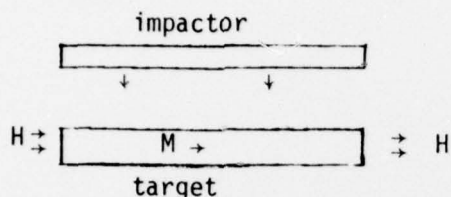


Figure 8. Typical experimental arrangement.

Figure 8 illustrates the typical arrangement of the type of experiment being discussed here. The field H can be induced by a permanent magnet or by having the target be an integral part of an electromagnet. If it is imagined that the magnetostrictive effect is associated with a stress in the direction of M in Fig. 8, then

$$\frac{1}{V_0} \left. \frac{\partial V_\alpha}{\partial H} \right|_0 = (1-2\nu) \left[\frac{\partial}{\partial H} \left(\frac{\delta l}{l} \right) \right]_0 \quad (129)$$

where ν is Poisson's ratio²⁰, and $(\delta l/l)$ is the strain in the M direction. For brevity let $\nu = 1/3$.

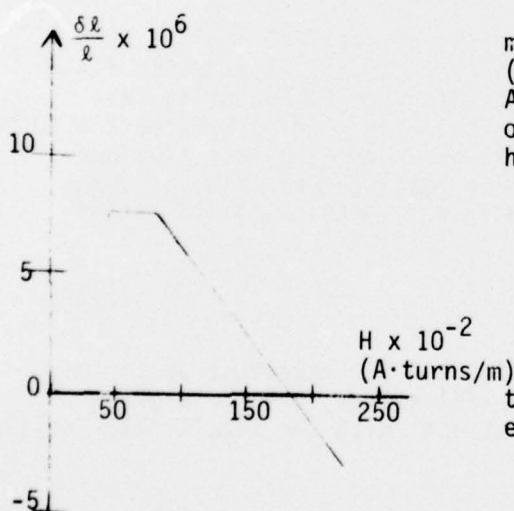


Figure 9. A crude approximation to the magnetostriction data for iron.

Figure 9 crudely illustrates magnetostriction data for iron (from Mattiat⁴⁹). For the 2000 Ampere-turn/m external field value of Curran and Hornemann we thus have, upon using Eq. (129),

$$\frac{H}{V_0} \left. \frac{\partial V_\alpha}{\partial H} \right|_0 = \frac{7}{3} \times 10^{-6}. \quad (130)$$

Letting δP_2 be the contribution to δP from the magnetostrictive effect

$$\delta P_2 \equiv - \left(\frac{V_0}{\Delta V_0} \right) \frac{HP_c}{V_0} \left. \frac{\partial V_\alpha}{\partial H} \right|_0 \quad (131)$$

$$\therefore \delta P_2 = - \frac{2.3 \times 10^{-6}}{5.1 \times 10^{-2}} P_c = - 5.9 \times 10^{-3} \text{ kb.} \quad (132)$$

The most interesting aspect of δP_2 of Eq. (132) is its sign; the sign of δP_2 indicates a decrease in transition pressure in the presence of a magnetic field, in agreement with the experiments of Curran and Hornemann. Of course Curran and Hornemann observed a decrease in transition pressure of approximately 10 kbar, while our δP_2 value is three orders of magnitude smaller than the mentioned experimental observations.

The most drastic assumption made in arriving at Eq. (132) is probably that of stress independent magnetostriction. The applied magnetic field as well as the shock wave each have an orientation effect upon the magnetic domain structure in an iron sample. While it is difficult to imagine how such an orientation interaction can effect δP_2 by three orders of magnitude, that possibility should not be immediately ruled out.

Barker and Hollenbach's⁴⁸ experiments were carried out at with an external field of $H = 2000$ Gauss, and observed $\delta P = 0$ to within 1 kbar. But 2000 Gauss is equal to 1.6×10^5 Ampere-turns per meter, and thus represents a magnetic field approximately two orders of magnitude larger than that employed by Curran and Hornemann. It is most interesting to note that, at least as far as the magnetostriction effect is concerned (see Fig. 9), the different magnetic field magnitudes give potentially radically different experiments. It thus seems that further experiment and theory must be carried out before this issue can be resolved. There is an applied motivation for such further work; a $\delta P > 10$ kbar could play a role in the further engineering development of tank armor; for a significant δP one could envision a tank commander flipping a switch in order to magnetize his vehicle just prior to an expected fragment impact.

V. CONCLUDING DISCUSSION

This report has been concerned with the interplay between phase change effects and shock wave physics. The report represents a summary of some beginning ideas on the subject.

Two motivated problems have been of main concern. The study of shock induced structural phase transitions in metallic alloys was

motivated by a desire to obtain a more efficient conversion medium for use in an optically activated detonator. The consideration of the effect of an applied external magnetic field on the $\alpha \rightarrow \epsilon$ phase transition in iron was motivated by a desire to investigate the role of some esoteric solid state concepts on the vulnerability of armor.

For the optically activated detonator problem our specific interest was in the possibility that the shock induced transition might result in avalanching of the amplitude of the propagating shock wave. Both macroscopic and microscopic considerations are shown to be consistent with the possible existence of such avalanching. In order to reach more definite conclusions further work is necessary. That further work includes more rigorous modeling of the macro and micro aspects of transition physics as well as the seeking of solutions of equations which are in the form of Eq. (102). The more rigorous modeling includes replacing the assumption of Eq. (68) with better physics, and attempting to explicitly include shear (wave) aspects of the transformation. The displacements associated with the room temperature transition in NiTi correspond to shear modes, and the one dimensional approach presented in this report has ignored that very important fact.

Although the report predicts a very small magnetoelastic contribution (of the order of 10^{-2} kbar) to the $\alpha \rightarrow \epsilon$ transition pressure in iron, that prediction is very preliminary. Considerably more work remains to be done in this problem area, for both magnetic field influences, and with respect to the possibility of tweaking other materials parameters which can be important to transformation physics.

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